Does Determinism conflict with Wave-Particle Realism? Proposal for an Experimental Test

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Wave-particle duality, superposition and entanglement are among the most counterintuitive aspects of quantum theory [1, 2]. Their clash with classical intuition motivated construction of hidden variable (HV) theories designed to remove or explain these "strange" quantum features, and contributed to the development of quantum technologies [2–4]. These technologies enable experiments with quantum controlling devices [5, 6], allowing, e.g., a freedom in temporal ordering of the control and detection and forcing modification [6–8] of the complementarity principle [9]. Here we study the entanglement-assisted delayed-choice experiment [10, 11] and show that, in the absence of superluminal communication, realism (defined as a property of photons being either particles or waves, but not both) is incompatible with determinism, even if the HV theory satisfying one of these properties reproduces quantum predictions. Our analysis does not use inequalities and is robust against experimental inefficiencies. We outline an experimental design that will be used to test our results.

The introduction of quantum controlling devices into well-known *gedanken* experiments provides new insight into quantum foundations [6]. A familiar example, Wheeler's delayed-choice experiment (WDC) [12–14] demonstrates wave-particle duality (Figure 1). The delayed-choice experiment with quantum control (Figure 2a) also highlights the complexity of space-time ordering of events, once parts of the experimental set-up become quantum systems [6, 11]. The quantum-controlled delayed-choice experiment has been recently implemented in several different systems [7, 11, 15–18]. A logical development is to ensure the quantumness of the controlling device either by means of a Bell-type test [18] or directly by using an entangled ancilla [10, 11].

The complementarity of setups [9] obscures a simultaneous presence of both wave and particle properties and allows a ("realist") view, i.e., at any moment of time, a photon is *either* a particle *or* a wave. The WDC experiment uncovers the difficulty inherent in this view by randomly choosing whether or not to insert the second beamsplitter after the photon enters the interferometer (Figure 1a). This delayed choice prevents a possible causal link between the experimental setup and the photon's behaviour: the photon should not "know" beforehand if it has to behave like a particle or like a wave.

We analyze the full entanglement control set-up in which the investigated photon and the entangled-pair ancilla are implemented by three different particles (Figure 2(b)). Our analysis reveals a new and surprising feature: classically appealing requirements of *realism* (photons are either particles or waves) and *determinism* (knowledge of HV determines the outcomes of all tests) are incompatible, even if the settings of the measurement devices are allowed to influence the hidden variables.

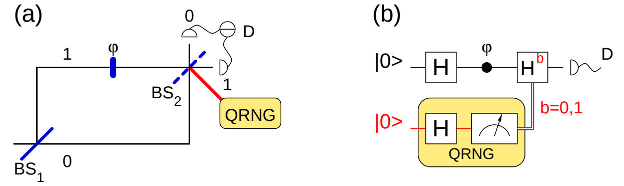

FIG. 1: Wheeler's delayed choice experiment: (a) The second beamsplitter is inserted or removed after the photon is inside the interferometer; this prevents the photon to "change its mind" about being a particle or a wave. The detectors observe either an interference pattern depending on the phase φ (wave behaviour), or an equal distribution of hits (particle behaviour). A quantum random number generator (QRNG) determines whether BS_2 is inserted or not. (b) The equivalent quantum network. The QRNG is an auxiliary quantum system, initially prepared in the superposition of 0 and 1 states and then measured. In the language of quantum networks a beamsplitter is represented by the Hadamard gate (the boxed H).

We use the notation of [6, 19]. We denote by $q(a, b, \dots)$ the quantum-mechanical probability distributions and by $p(a, \dots, \lambda, \dots)$, $p(a, \dots|\lambda, \dots)$ the (conditional) predictions of HV theories with a hidden variable λ.

The system we analyze consists of three qubits: a photon A and a maximally entangled ancilla, B and C (Fig. 2b). We denote the measurement outcomes for the photon as a, and for the two ancilla qubits (the EPR pair) as b and c; the corresponding detectors are D_a, D_b and D_c. The system is prepared in the initial state $\frac{1}{\sqrt{2}}|0\rangle_A\big(|00\rangle + |11\rangle\big)_{BC}$. Photon A is incident on a Mach-Zehnder interferometer (MZI) in

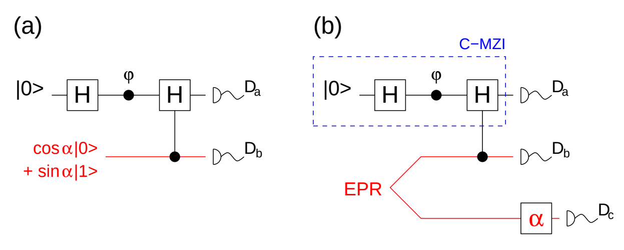

FIG. 2: Quantum networks for delayed-choice experiments: (a) delayed-choice with a quantum control [6]; (b) entanglement-assisted quantum delayed-choice experiment [10, 11]. The ancilla C is measured along the direction $-\alpha$, equivalent to the application of a rotation $R_y(\alpha) = e^{i\alpha\sigma_y}$ before a measurement in the computational basis.

which the second beamsplitter is quantum-controlled by qubit B of the EPR pair. The third qubit C undergoes a σ_y rotation $R_y(\alpha) = e^{i\alpha\sigma_y}$ followed by a measurement in the computational basis. The state before the final detection is

$$\begin{aligned} |\psi\rangle_{ABC} &= \tfrac{1}{\sqrt{2}}\big(\cos\alpha|\mathsf{p}\rangle_A|0\rangle_B + \sin\alpha|\mathsf{w}\rangle_A|1\rangle_B\big)|0\rangle_C \\ &- \tfrac{1}{\sqrt{2}}\big(\sin\alpha|\mathsf{p}\rangle_A|0\rangle_B - \cos\alpha|\mathsf{w}\rangle_A|1\rangle_B\big)|1\rangle_C \end{aligned} \tag{1}$$

where the wavefunctions $|\mathsf{p}\rangle = \frac{1}{\sqrt{2}}(|0\rangle + e^{i\varphi}|1\rangle)$ and $|\mathsf{w}\rangle = e^{i\varphi/2}(\cos\frac{\varphi}{2}|0\rangle - i\sin\frac{\varphi}{2}|1\rangle)$ describe particle and wave behaviour, respectively [6]. From the state (1) one can calculate the quantum statistics $q(a,b,c)$ for the observables a, b, c, from which all the other probability distributions follow.

Our strategy is to show that $q(a,b,c)$ cannot result from a probability distribution $p(a,b,c,...)$ of a hidden-variable theory satisfying the requirements of realism (of particle and wave properties) and determinism. Any such HV theory should satisfy the following conditions:
(i) *Adequacy*: a HV theory should reproduce the experimentally observed quantum mechanical (QM) statistics. Mathematically, this means that the QM probability is given by summing/integrating over all hidden variables (λ, Λ):

$$q(a,b,c) = \sum_{\lambda,\Lambda} p(a,b,c,\lambda,\Lambda) = \sum_{\lambda,\Lambda} p(a,b,c|\lambda,\Lambda)p(\lambda,\Lambda). \tag{2}$$

(ii) *Realism of particle and wave properties*: for a given photon we require the property of being a "particle" or a "wave" to be intrinsic, i.e., to be unchanged during its lifetime. This condition is essential and selects from the set of HV theories reproducing the QM statistics those models that have meaningful notions of "particle" and "wave" [6]. With this in mind, we single out a dichotomic hidden variable λ which determines if a given photon is a particle, $\lambda = \mathsf{p}$, or a wave, $\lambda = \mathsf{w}$. A *particle* in an open interferometer ($b = 0$) is insensitive to the phase shift in one of the arms and therefore has the statistics

$$p(a|b=0,\lambda=\mathsf{p},\Lambda) = p(a|b=0,\lambda=\mathsf{p}) = \big(\tfrac{1}{2},\tfrac{1}{2}\big). \tag{3}$$

By contrast, a *wave* in a closed MZI ($b = 1$) shows interference

$$p(a|b=1,\lambda=\mathsf{w},\Lambda) = p(a|b=1,\lambda=\mathsf{w}) = \big(\cos^2\tfrac{\varphi}{2},\sin^2\tfrac{\varphi}{2}\big). \tag{4}$$

As a concession to HV theories the behaviour of a particle in a closed MZI and of a wave in an open one are not constrained. The remaining hidden variables of the theory will be collectively denoted by Λ. For simplicity we assume Λ is discrete; the analysis can be easily generalized to the continuous case.

Since our set-up contains an entangled pair, we also add a standard assumption of HV theories, namely
(iii) *Strong determinism* [19]: a complete knowledge of the hidden variables Λ (and, perhaps, λ) determines the individual outcomes of D_c and D_b. Specifically,

$$p(c|\lambda,\Lambda) = \delta_{c,h(\lambda,\Lambda)}, \tag{5}$$

where h is some binary-valued function of the HV. This is equivalent to a decomposition of the hidden-variable space $\mathcal{L}$ into complementary sets $\mathcal{L}_0$ and $\mathcal{L}_1$, such that for $(\lambda,\Lambda) \in \mathcal{L}_0$ $(\mathcal{L}_1)$ the detector D_c always measures $c = 0$ $(c = 1)$.

An alternative to (iii) is a requirement of weak determinism, which asserts that a complete knowledge of HV determines the outcomes of D_b and D_c jointly. Using the weak determinism condition does not alter our conclusions, but we prefer strong determinism in order to simplify the exposition.

The usual quantum delayed-choice experiment can be seen as a particular case. Then [6] it is possible to construct an HV theory which in addition does not modify the notion of particles and waves by having a perfect correlation between the outcomes of D_b and the HV λ. However, in order to reproduce the observed statistics, the HV probability distribution $p(\lambda) = (f, 1-f)$ is fully determined, in this model, by the bias parameter α. Such an HV theory must be dismissed if one is unwilling to accept the necessary deep conspiratorial correlations [20].

Replacing the ancilla with an entangled pair allows one to combine quantum control and a possibility for space-like separation of the events. As discussed in the Methods, the only non-trivial condition for preserving the identity of waves and particles is

$$\sum_{(\lambda,\Lambda)\in\mathcal{L}_0} p(\lambda,\Lambda) = \tfrac{1}{2}\cos^2\alpha. \tag{6}$$

Hence without the additional restrictions it is trivial to construct an HV theory satisfying the conditions (i)-(iii).

These solutions are ruled out if we accept non-signalling, i.e., that information propagates slower than the speed of light. There are two options of "denial of information": either the photon A and the EPR pair can be prepared independently, or a decision about applying the rotation $R_y(\alpha)$ on C is made after the three photons are produced. Each option by itself is sufficient to rule out HV theories.

If the photon A and the EPR pair BC are prepared independently, then the HV are generated independently, i.e.,

$$p(\mathsf{p},\Lambda) = fF(\Lambda), \qquad p(\mathsf{w},\Lambda) = (1-f)F(\Lambda), \tag{7}$$

for some probability distributions $(f, 1-f)$ for λ and $\{F(\Lambda)\}$ for Λ. Unlike the typical Bell-inequality scenarios, we have a single measurement set-up which involves two independent HV distributions. We show in the Methods that in this case Eq. (6) leads to the contradictory conditions

$$f = \cos^2\alpha = \sin^2\alpha, \tag{8}$$

which are impossible to satisfy for a generic rotation α.

Furthermore, even if the production of photons is not independent, a delay in the decision about application or non-application of the rotation α until after the production (and even more spectacularly, the detections D_a and D_b), makes the solution of Eq. (6) impossible.

The proposed experimental set-up is shown in Fig. 3. Two pump pulses (blue) are incident on two nonlinear crystals and generate via spontaneous parametric down-conversion (SPDC) two pairs of entangled photons (red). One of the photons is the trigger and the other three are the computational

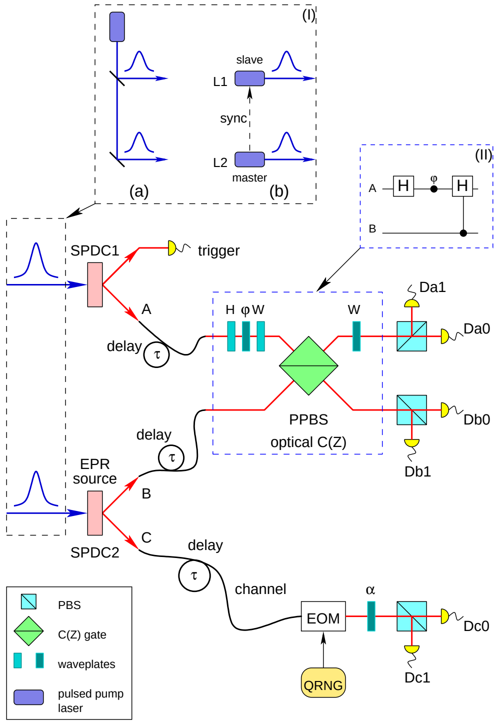

FIG. 3: Proposed experimental setup. Two pump pulses (blue) generate, via SPDC, two pairs of entangled photons (red). The first photon is the trigger and the remaining three the computational qubits A, B, C, with B, C the EPR pair. Inset (I): the two pump pulses can be produced by either a single laser (a), or by two independent and synchronised lasers (b). Inset (II): the quantum-controlled Mach-Zehnder interferometer. The optical delays in the three photon arms, τ_A, τ_B, τ_C ensure that the three photons are measured at the same time, and depend on the exact spacing of the labs and the length of the optical fibres.

qubits A, B, C, with B and C the EPR pair. This is in contrast with [11] where only two particles are used to encode three qubits.

Photons A and B are held in the lab (with appropriate delay lines) and together they implement the controlled MZI. The central element is the quantum switch, i.e., the controlled-Hadamard gate $C(H)$. The photonic $C(Z)$ gate is implemented using a partially-polarizing beam-splitter (PPBS) and therefore is performed probabilistically via post-selection [21, 22]. The single-qubit rotations on photon A (the gates H, φ and W) are enacted with optical wave-plates.

Photon C is sent through a channel at a distant location x_c, then measured in a rotated basis. In the ideal case two independent and synchronised lasers would generate the two photon pairs (Fig.3, inset I(b), [23, 24]); in this case we can use Eq. (7) to describe independent probability distributions for λ and Λ. The choice of $\alpha \neq \pi/4$ would not allow Eq. (8) to be fulfilled, but the actual experiment might include tests with a range of α.

However, typical experiments with multiple photons use a single pump laser and the distributions could be correlated. In this case the paradox will be ensured by enforcing a strict space-like separation between the random choice of $R_y(\alpha)$ and the quantum switch. The random choice between at least two settings α_1, α_2 is delayed until the photon A is inside the interferometer. The fast choice between different settings can be performed with an electro-optical modulator (EOM) controlled by a fast quantum number generator (QRNG) [25].

To conclude, we show that while the statistics on the EPR pair alone can be reproduced by a deterministic HV theory, adding local realism in the form of photons being either a wave or particle independently of causally disconnected events makes the classical intuition untenable. We presented a feasible experiment which could test this contradiction.

Methods

Theoretical analysis. We assume the standard probability rules for marginal $p(i) = \sum_j p(i,j)$ and conditional probabilities $p(i,j) = p(i|j)p(j) = p(j|i)p(i)$. Using the conditional probability rule for p, the first requirement, Eq.(2), can be expressed as:

$$q(a,b,c) = \sum_{\lambda,\Lambda} p(a|b,c,\lambda,\Lambda)p(b|c,\lambda,\Lambda)p(c|\lambda,\Lambda)p(\lambda,\Lambda). \quad (9)$$

We introduce the following notation for $p(\lambda,\Lambda)$. We label the additional hidden variables by a discrete index i, such that the hidden variables take the values (p,Λ^i) and (w,Λ^i), with the prior probabilities for HV as $f^i_{\mathsf{p}}, f^i_{\mathsf{w}}, \ldots$, corresponding to the photon being a particle (wave). If necessary, we refine the list of HV Λ^i in such a way that the set $\mathcal{L}_c$ consists of the hidden variables (λ,Λ^i), $i \in I_c$, that result in the outcome $c = 0, 1$. To maintain this convention some of the prior probabilities f^i_λ can be set to zero.

Any HV theory which matches the quantum predictions satisfies

$$q(a,b,c=0) = \sum_{(\lambda,\Lambda)\in\mathcal{L}_0} p(a|b,\lambda,\Lambda)p(b|\lambda,\Lambda)p(\lambda,\Lambda), \quad (10)$$

with an analogous expression for $c = 1$. The probability of the outcome $c = 0$ satisfies

$$q_0 \equiv q(c=0) = \sum_{(\lambda,\Lambda)\in\mathcal{L}_0} p(\lambda,\Lambda) = \sum_{i\in I_0} f^i_{\mathsf{p}} + f^i_{\mathsf{w}}. \quad (11)$$

For the controlling pair in a maximally entangled state $q_0 = \frac{1}{2}$. Conservation of probability implies $\sum_i f^i_{\mathsf{p}} + f^i_{\mathsf{w}} = 1$.

The requirement of strong determinism for the results of D_b will be imposed later. Note that single-qubit measurements [26], as well as tests on the entangled pairs in an arbitrary fixed set-up, can be successfully modelled by strongly deterministic HV theories [20].

The second requirement (ii) provides a definition of particles and waves in an HV theory according to their behaviour in a Mach-Zehnder interferometer [6], as given by Eqs. (3) and (4). Two other conditional probabilities specify a putative behaviour of a wave ($\lambda = \mathsf{w}$) in a open ($b = 0$) interferometer and of a particle ($\lambda = \mathsf{p}$) in a closed ($b = 1$) one. We denote these two unknown distributions by x and y, respectively

$$p(a|b=0,\lambda=\mathsf{w},\Lambda=\Lambda^i) = (x_i, 1-x_i), \quad (12)$$
$$p(a|b=1,\lambda=\mathsf{p},\Lambda=\Lambda^i) = (y_i, 1-y_i), \quad (13)$$

i.e., we distinguish between $\lambda = \mathsf{w}$ and $\lambda = \mathsf{p}$ cases.

The remaining two sets of variables are the probability distributions for b conditioned on the values of hidden variables λ and Λ:

$$p(b|\lambda=\mathsf{p},\Lambda=\Lambda^i) = (z_i, 1-z_i) \quad (14)$$
$$p(b|\lambda=\mathsf{w},\Lambda=\Lambda^i) = (v_i, 1-v_i). \quad (15)$$

The strong determinism assumption implies that all the variables z_i and v_i are either 0 or 1.

By imposing condition (i), i.e., the HV theory reproduces the QM probability $q(a,b,c)$, we generalise equations (10)–(12) in ref. [6]:

$$\sum_{i\in I_0}(x_i-\tfrac{1}{2})v_i f^i_{\mathsf{w}}=0, \tag{16a}$$

$$\sum_{i\in I_1}(x_i-\tfrac{1}{2})v_i f^i_{\mathsf{w}}=0, \tag{16b}$$

and

$$\sum_{i\in I_0}\left(z_i f^i_{\mathsf{p}}+v_i f^i_{\mathsf{w}}\right)=q_0\cos^2\alpha, \tag{17a}$$

$$\sum_{i\in I_1}\left(z_i f^i_{\mathsf{p}}+v_i f^i_{\mathsf{w}}\right)=(1-q_0)\sin^2\alpha, \tag{17b}$$

are obtained for $b=0$ case, and the last independent pair corresponds to $a=0$, $b=1$ case,

$$\cos^2\tfrac{\varphi}{2}\left(q_0-\sum_{i\in I_0}f^i_{\mathsf{w}}\right) = \sum_{i\in I_0}f^i_{\mathsf{p}}\big(z_i\cos^2\tfrac{\varphi}{2}+y_i(1-z_i)\big) \tag{18a}$$

$$\cos^2\tfrac{\varphi}{2}\left(1-q_0-\sum_{i\in I_1}f^i_{\mathsf{w}}\right) = \sum_{i\in I_1}f^i_{\mathsf{p}}\big(z_i\cos^2\tfrac{\varphi}{2}+y_i(1-z_i)\big), \tag{18b}$$

The parameters z_i, v_i describe the HV theory in question and are independent of the settings the experimenter controls. On the other hand, the probabilities f^i_λ may depend on the values of φ, α and q_0. From Eq. (16a) we see that to keep the parameters x_i undetermined (i.e., do not prescribe statistics for behaviour of "waves" in the open interferometer), all factors $v_i f^i_{\mathsf{w}}$ should be zero individually, and to keep the parameters y_i undetermined, the interferometer should be always open when the photon is a "particle", i.e.

$$v_i f^i_{\mathsf{w}}=0, \qquad z_i=1, \qquad \forall i. \tag{19}$$

As a result, the third pair of equations is identically satisfied and the only remaining equations are

$$\sum_{i\in I_0}f^i_{\mathsf{p}}=q_0\cos^2\alpha, \qquad \sum_{i\in I_1}f^i_{\mathsf{p}}=(1-q_0)\sin^2\alpha, \tag{20}$$

reducing to Eq. (6) for $q_0=\frac{1}{2}$.

Using the product form of the prior probability distribution of Eq. (7), Eqs. (11) and (20) (with $q_0=\frac{1}{2}$) one reaches the contradiction of Eq. (8).

Acknowledgments

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